aircraft concerned. Such limits could easily be applied to the calculated performance. Also the effect of flap has been considered only at set positions; if the aircraft has infinitely variable flap setting, then use of flap and attitude for descent control is possible. There are further possibilities of incorporating such control in a CCV mode.

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Added Mass and the Dynamic Stability of Parachutes

John A. Eaton*
Edgley Aircraft Ltd., Cambridge, England

Introduction

OVER the last two decades advances in the modeling of parachute dynamics and stability have been concentrated in the simulation of the solid body components of systems with ever-increasing numbers of degrees of freedom, made possible by the availability of ever-more powerful computers. In contrast, the fluid components of parachute systems have long languished for want of attention. One of the basic fluid components is the subject of this Note, which reviews the implementation of the added fluid mass in stability studies of parachutes. The complete form of the added mass tensor for a rigid axisymmetric parachute is obtained and implemented correctly for the first time, and results[1] indicate that added mass effects are even more significant than hitherto predicted.[2-5] The equations of motion in the Tory and Ayres model[5] are shown to be incorrect for the assumptions adopted.

Added Mass in the Equations of Motion

Consider a dynamical system consisting of an arbitrary body moving in an arbitrary (real or ideal) fluid. Choose an orthogonal Cartesian body axis set $Oxyz$ and let P_i, H_i, $(i=1,2,3)$ be the component linear momenta and the component angular momenta of the system referred to $Oxyz$. The origin inertial velocity and angular velocity components are V_i and ω_i, and the external forces and moments acting on the system are F_i and M_i. The general form of the Euler equations of motion for the complete system may then be written as

$$F_i = \dot{P}_i + \epsilon_{ijk}\omega_j P_k \tag{1}$$

$$M_i = \dot{H}_i + \epsilon_{ijk} V_j P_k + \epsilon_{ijk}\omega_j H_k \tag{2}$$

Using Hamilton's principle, Kirchhoff[6] showed in detail how, for motion of a rigid body in an ideal fluid, the external forces and moments on the body due to the fluid inertia may be derived from the added mass tensor and the generalized velocities of the body coordinate origin. Analogous to the solid body inertia tensor ($[B]^7$), principal axes, and mass center, there exist an added inertia tensor ($[A]$), principal axes, and a center of added mass. Referring to Ibrahim,[7] we find the equivalent external forces and moments exerted on the fluid [Ref. 7, Eqs. (3.46) and (3.47)] are

$$F_{F_i} = A_{i\alpha}\dot{V}_\alpha + \epsilon_{ijk}\omega_j A_{k\alpha} V_\alpha \tag{3}$$

$$M_{F_i} = A_{i+3,\alpha}\dot{V}_\alpha + \epsilon_{ijk} V_j A_{k\alpha} V_\alpha + \epsilon_{ijk}\omega_j A_{k+3,\alpha} V_\alpha \tag{4}$$

where

$$A_{i\alpha}\dot{V}_\alpha = A_{ij}\dot{V}_j + A_{i,j+3}\dot{\omega}_j \qquad (\alpha = 1,\ldots,6;\ i,j = 1,2,3)$$

$$A_{k\alpha} V_\alpha = A_{kl} V_l + A_{k,l+3}\omega_l \qquad (k,l = 1,2,3)$$

If B_{ij} is of dynamic significance, the corresponding A_{ij} will be significant when its inertia or mass ratio $\mu_{ij} = A_{ij}/B_{ij} \geq O(1)$.

Previous Representations of the Added Mass Tensor

For the basic stability modeling of a parachute with a fully deployed canopy, conventional practice has been to assume the canopy to be rigid and axisymmetric and the fluid to be inviscid, irrotational, and incompressible.

Henn[8] was one of the first to take added mass into account in a planar, three degree-of-freedom study of parachute stability. Replacing the canopy with a rigid ellipsoidal, air-filled body, his added mass components comprised contributions from the fluid regions interior ("included mass" and "included moment of inertia") and exterior ("additional apparent" masses and moment of inertia) to the body. In the present notation (see next section) his added mass components were A_{11}, A_{33}, and A_{55}. By analytical solution of the three linearized equations of motion, Henn demonstrated strong influences on the damping and frequency of lateral oscillations (of a mass-geometrically typical personnel parachute) due to individual variations of A_{11} and A_{33} within the ranges $\mu_{11} = 0.0\text{-}0.6$ and $\mu_{33} = 0.0\text{-}1.0$ for baseline values of $\mu_{11} = \mu_{33} = 0.5$.

Henn's equations were widely used until 1962, when Lester[9] showed that they were erroneous; in uncritically applying to the added mass components the rigid-dynamical equations, rather than the fundamental Kirchhoff equations,[6] Henn had violated the concept of added mass.

Ludwig and Heins,[2] again for planar three degree-of-freedom motion, confined their added mass implementation to Henn's two isotropic "included" terms, but solved the nonlinear equations of motion using both analog and digital techniques. Using values of μ_{11} $(=\mu_{33})$ in the range of 0.6-1.4, they concluded that the added mass mainly affected oscillation amplitude and damping, had a minor effect on frequency, and overall was not very important.

White and Wolf,[3] using the same added mass components as Ludwig and Heins, solved the five degree-of-freedom equations of motion for a single rigid body system and showed that stability decreased with increasing μ.

One exception to the trend of reducing the representation of added mass was the six degree-of-freedom rigid body model introduced in 1972 by Tory and Ayres,[5] where, on the basis of the dubious assumption of a real (physical) distinction between "included" and "apparent" mass,[10] they reverted to Henn's original collection of components. As part of the overall model validation[1] the author carried out a sensitivity analysis[4] for a typical personnel parachute system. This indicated that A_{33} and A_{55} were not important in regard to stability, but that A_{11} had a significant effect on damping. Damping increased with increasing store mass, as found by White and Wolf, but was unaffected by A_{33}. Baseline mass ratio values were used μ_{11} $(=\mu_{22}) = 0.5$, $\mu_{33} = 0.9$, and the ranges covered were $\mu_{11} = 0.0\text{-}6.7$ and $\mu_{33} = 0.0\text{-}6.5$.

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*Consulting Engineer. Member AIAA.

Byushgens and Shilov,[11] in a purely analytical three degree-of-freedom study, pointed out inadequacies of added mass modeling in previous models.[2] Their three degree-of-freedom equations of motion implement added mass in a manner identical to that of Lester, of whose work they appear to be unaware.

Present Implementation

Discovery of Ref. 11 in early 1980 prompted the author to check the added mass terms in Tory and Ayres' model; their equations of motion were found to be erroneous, having suffered, to a greater degree than Henn's, from application of the rigid dynamical equations.

We apply Eqs. (1-4) to the particular case of a rigid parachute of axisymmetric shape moving in an ideal fluid.[1] Body axes are chosen so that Oz is on the axis of symmetry and Ox and Oy are parallel to the two other principal body axes. The body mass m comprises the canopy, rigging lines, and store masses (m_i) with mass centers located at z_i, respectively. First, for the solid body components, either from Eqs. (1) and (2) or from Eqs. (3) and (4) by substituting B_{ij} for A_{ij} (the algebra involved is the same), with

$$K_1 = \sum_{i=1}^{n} m_i z_i$$

$$F_B = \begin{bmatrix} m(\dot{u}-vr+wq)+K_1(\dot{q}+rp) \\ m(\dot{v}-wp+ur)-K_1(\dot{p}-qr) \\ m(\dot{w}-uq+vp)-K_1(p^2+q^2) \end{bmatrix} \quad (5)$$

$$M_B = \begin{bmatrix} I_{xx}\dot{p}-K_1(\dot{v}-wp+ur)-(I_{yy}-I_{zz})qr \\ I_{yy}\dot{q}+K_1(\dot{u}+wq-vr)+(I_{xx}-I_{zz})pr \\ I_{zz}\dot{r}+(I_{yy}-I_{xx})pq \end{bmatrix} \quad (6)$$

Kirchhoff has shown that for the axisymmetric body shape, with the body origin O located at some point on the axis of symmetry, only four different added mass components are obtained. These are A_{11}, A_{33}, A_{55}, and A_{15}, since from symmetry $A_{22}=A_{11}$, $A_{44}=A_{55}$, $A_{51}=A_{15}$, $A_{42}=A_{24}$, and $A_{24}=-A_{15}$. Also, by shifting the body origin along Oz by a distance $a=-A_{15}/A_{11}$, the components $A_{15}=-A_{24}=0$ and only three unique components remain (Ref. 6, p. 251).

Using these, and expanding Eqs. (3) and (4) we find

$$F = \begin{bmatrix} A_{11}(\dot{u}-vr)+A_{33}wq+A_{15}(\dot{q}+rp) \\ A_{11}(\dot{v}+ur)-A_{33}wp-A_{15}(\dot{p}-qr) \\ A_{33}\dot{w}-A_{11}(uq-vp)-A_{15}(p^2+q^2) \end{bmatrix} \quad (7)$$

$$M_F = \begin{bmatrix} A_{55}\dot{p}-A_{15}(\dot{v}-wp+ur)-A_{55}qr+(A_{33}-A_{11})vw \\ A_{55}\dot{q}+A_{15}(\dot{u}+wq-vr)+A_{55}pr-(A_{33}-A_{11})uw \\ 0 \end{bmatrix} \quad (8)$$

Combining Eqs. (5) and (7), (6) and (8), the full equations of motion are

$$F = \begin{bmatrix} (m+A_{11})(\dot{u}-vr)+(m+A_{33})wq+(K_1+A_{15})(\dot{q}+rp) \\ (m+A_{11})(\dot{v}+ur)-(m+A_{33})wp-(K_1+A_{15})(\dot{p}-qr) \\ (m+A_{33})\dot{w}-(m+A_{11})(uq-vp)-(K_1+A_{15})(p^2+q^2) \end{bmatrix} \quad (9)$$

$$M = \begin{bmatrix} (I_{xx}+A_{55})\dot{p}-(K_1+A_{15})(\dot{v}-wp+ur)-(I_{yy}+A_{55}-I_{zz})qr+(A_{33}-A_{11})vw \\ (I_{yy}+A_{55})\dot{q}+(K_1+A_{15})(\dot{u}+wq-vr)+(I_{xx}+A_{55}-I_{zz})pr-(A_{33}-A_{11})uw \\ I_{zz}\dot{r}+(I_{yy}-I_{xx})pq \end{bmatrix} \quad (10)$$

Tory and Ayres assumed the added mass to be made up of an "included" air mass in the shape of an ellipsoid along with an "apparent" mass and "apparent moment of inertia" external to the ellipsoid. The mass of the ellipsoid was m_E, with moments of inertia $I_{xx_E}=I_{yy_E}$, and I_{zz_E}; the apparent masses were $K_x m_E$ along x and y and $K_z m_E$ along z; the apparent moments of inertia were $K' I_{xx_E}$ about x and y. For an ellipsoid in an ideal fluid $A_{15}=A_{24}=0$. Thus, they had in effect

$$A_{11} = (1+K_x)m_E = A_{22}$$

$$A_{33} = (1+K_z)m_E$$

$$A_{55} = (1+K')I_{xx_E} = A_{44}$$

$$A_{66} = I_{zz_E}$$

from which they derived

$$F_F = \begin{bmatrix} A_{11}(\dot{u}-vr+wq) \\ A_{11}(\dot{v}-wp+ur) \\ A_{33}(\dot{w}-uq+vp) \end{bmatrix} \quad (11)$$

$$M_F = \begin{bmatrix} A_{55}(\dot{p}-qr)+A_{66}qr \\ A_{55}(\dot{q}+pr)-A_{66}pr \\ A_{66}\dot{r} \end{bmatrix} \quad (12)$$

Comparing Eqs. (7) and (11), (8) and (12), apart from the obvious differences, note the *steady-state* moment terms $(A_{33}-A_{11})vw$ and $-(A_{33}-A_{11})uw$ in the present equations; these are discussed in Ref. 1.

Simulation Results

Applying the full three-dimensional form of Eqs. (9) and (10) to a modified form of the Tory-Ayres computer program gave the first complete nonlinear solutions to the six degree-of-freedom equations of motion for a parachute with an anisotropic added mass tensor.[1]

The assessment technique used involves planar, large-disturbance conditions from which mean damping ratio and frequency measurements are derived (see Ref. 4 for details). Some comparative results are presented in Table 1 (θ is the Euler angle about the Oy axis). The sensitivity analysis of Ref. 4 used aerodynamic data for the aeroconical canopy and cases 3 and 4 show the currently much more powerful effect of μ_{11} in the damping. Well-defined drop test data[1] were obtained for validation purposes, these data being for the flat circular canopy. Case 6 shows that the present model is now strongly unstable with the baseline added mass values selected by Tory and Ayres[5] (experiments[12] suggest that $\mu_{33} \sim 0.4$ for this case). A hitherto ineffective pitch damping term 1_p[13] was used to prevent divergence (case 8). The very strong effect of μ_{11} on frequency and limit cycle amplitude is seen in cases 10 and 11.

Table 1 Comparison of added mass effects on the present and Tory/Ayres models (planar motion)

Case	Canopy/ model	μ_{11}[a]	μ_{33}[a]	l_p	Mean damping ratio ζ	Period, s	θ,[b] deg
	Aeroconical						
1	Tory/Ayres	0.50	0.80	0.00	0.15	4.7	(0)[c]
2	Present	0.50	0.80	0.00	0.00	4.4	35
3	Tory/Ayres	0.00	0.80	0.00	0.19	4.6	(0)
4	Present	0.00	0.80	0.00	~Critical damping		~ 0
	Flat circular						
5	Tory/Ayres	0.35	0.60	0.00	0.03	5.9	(0)
6	Present	0.35	0.60	0.00	Strong divergence		
7	Present	0.35	0.00	0.00	Steady glide		~0
8	Present	0.35	0.60	−0.08	0.00	4.9	83
9	Present	0.35	0.50	−0.08	0.07	6.1	(0)
10	Present	0.00	0.50	−0.08	0.00	3.2	44
11	Present	0.15	0.50	−0.08	0.00	4.0	58

[a] For $\mu_{55} = A_{55}/mz_s^2 \sim 0.01$; $\mu_{15} = 0$. [b] Amplitude in limit cycle. [c] Convergent.

The results of these numerical tests on the present flat circular model using baseline values $\mu_{11}(=\mu_{22}) = 0.2$ and $\mu_{33} = 0.4$, within the narrow ranges $\mu_{11} = 0.0$-0.5 and $\mu_{33} = 0.0$-1.0, indicate that both A_{11} and A_{33} are significant. In particular,

1) Damping *increases* with A_{11}.
2) Period increases with A_{11}.
3) Damping decreases with A_{33}.
4) Period decreases with A_{33}.

Thus A_{11} and A_{33} have opposite effects on parachute stability, so the correct implementation is important. This implies that much improved experimental values of the A_{ij} for parachutes are needed; the current order-of-magnitude estimates are very inadequate for stability studies on personnel-type, low-porosity systems.

Conclusions

The correct form of the added mass tensor for a rigid axisymmetric parachute in ideal flow has been implemented in a six degree-of-freedom computer model, and results indicate that added mass effects are much more significant than hitherto predicted. In particular, the component of added mass along the axis of symmetry has a strong influence on stability. Better experimental measures of added mass are needed.

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